HOMEWORK #4 SOLUTIONS

Problem #1: RDK 18.24

Solve: (a) The mean free path is

$$\lambda = \frac{1}{4\sqrt{2}\pi(N/V)r^2}$$

where $r \approx 0.5\times10^{-10}$ m is the atomic radius for helium and N/V is the gas number density. From the ideal-gas law,

$$\frac{N}{V} = \frac{p}{kT} = \frac{0.10\text{ atm}\times101{,}300\text{ Pa/atm}}{(1.38\times10^{-23}\text{ J/K})(10\text{ K})} = 7.34\times10^{25}\text{ m}^{-3}$$

$$\Rightarrow \lambda = \frac{1}{4\sqrt{2}\pi(7.34\times10^{25}\text{ m}^{-3})(0.5\times10^{-10}\text{ m})^2} = 3.1\times10^{-7}\text{ m} = 310\text{ nm}$$

(b) The root-mean-square speed is

$$v_{\text{rms}} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3(1.38\times10^{-23}\text{ J/K})(10\text{ K})}{4\times(1.661\times10^{-27}\text{ kg})}} = 250\text{ m/s}$$

where we used $A = 4$ u as the atomic mass of helium.

(c) The average energy per atom is $e_{\text{avg}} = \frac{3}{2}k_BT = \frac{3}{2}(1.38\times10^{-23}\text{ J/K})(10\text{ K}) = 2.1\times10^{-22}$ J.

Problem #2: RDK 18.32

Solve: The conservation of energy equation $(\Delta E_{\text{th}})_{\text{gas}} + (\Delta E_{\text{th}})_{\text{solid}} = 0$ J is

$$n_{\text{gas}}(C_V)_{\text{gas}}(T_f - T)_{\text{gas}} + n_{\text{solid}}(C_V)_{\text{solid}}(T_f - T_i)_{\text{solid}} = 0\text{ J}$$

$$\Rightarrow (1.0\text{ mol})(12.5\text{ J/mol K})(-50\text{ K}) + (1.0\text{ mol})(25.0\text{ J/mol K})(\Delta T)_{\text{solid}} = 0 \Rightarrow (\Delta T)_{\text{solid}} = 25°\text{C}$$

The temperature of the solid increases by 25°C.

Problem #3: RDK 18.34

Visualize: At low temperatures, $C_v = \frac{3}{2}R = 12.5$ J/mol K. At room temperature and modestly hot temperatures, $C_v = \frac{5}{2}R = 20.8$ J/mol K. At very hot temperatures, $C_v = \frac{7}{2}R = 29.1$ J/mol K.

Solve: (a) The number of moles of diatomic hydrogen gas in the rigid container is

$$\frac{0.20\text{ g}}{2\text{ g/mol}} = 0.10\text{ mol}$$

The heat needed to change the temperature of the gas from 50 K to 100 K at constant volume is

$$Q = \Delta E_{\text{th}} = nC_v\Delta T = (0.10\text{ mol})(12.5\text{ J/mol K})(100\text{ K} - 50\text{ K}) = 62\text{ J}$$

(b) To raise the temperature from 250 K to 300 K,

$$Q = \Delta E_{\text{th}} = (0.10\text{ mol})(20.8\text{ J/mol K})(300\text{ K} - 250\text{ K}) = 100\text{ J}$$

(c) To raise the temperature from 2250 K to 2300 K, $Q = \Delta E_{\text{th}} = nC_v\Delta T = (0.10\text{ mol})(29.1\text{ J/mol K})(50\text{ K}) = 150$ J

Problem #4: RDK 18.48

Solve: **(a)** The number of molecules of helium is

$$N_{\text{helium}} = \frac{pV}{k_B T} = \frac{(2.0\times1.013\times10^5\ \text{Pa})(100\times10^{-6}\ \text{m}^3)}{(1.38\times10^{-23}\ \text{J/K})(373\ \text{K})} = 3.936\times10^{21}$$

$$\Rightarrow n_{\text{helium}} = \frac{3.936\times10^{21}}{6.022\times10^{23}\ \text{mol}^{-1}} = 6.536\times10^{-3}\ \text{mol}$$

The initial internal energy of helium is

$$E_{\text{helium i}} = \tfrac{3}{2} N_{\text{helium}} k_B T = 30.4\ \text{J} \approx 30\ \text{J}$$

The number of molecules of argon is

$$N_{\text{argon}} = \frac{pV}{k_B T} = \frac{(4.0\times1.013\times10^5\ \text{Pa})(200\times10^{-6}\ \text{m}^3)}{(1.38\times10^{-23}\ \text{J/K})(673\ \text{K})} = 8.726\times10^{21}$$

$$\Rightarrow n_{\text{argon}} = \frac{8.726\times10^{21}}{6.022\times10^{23}\ \text{mol}^{-1}} = 1.449\times10^{-2}\ \text{mol}$$

The initial thermal energy of argon is

$$E_{\text{argon i}} = \tfrac{3}{2} N_{\text{argon}} k_B T = 121.6\ \text{J} \approx 122\ \text{J}$$

(b) The equilibrium condition for monatomic gases is

$$(e_{\text{helium f}})_{\text{avg}} = (e_{\text{argon f}})_{\text{avg}} = (e_{\text{total}})_{\text{avg}}$$

$$\Rightarrow \frac{E_{\text{helium f}}}{n_{\text{helium}}} = \frac{E_{\text{argon f}}}{n_{\text{argon}}} = \frac{E_{\text{tot}}}{n_{\text{tot}}} = \frac{(30.4+121.6)\ \text{J}}{(6.54\times10^{-3}+1.449\times10^{-2})\ \text{mol}} = 7228\ \text{J/mol}$$

$$\Rightarrow E_{\text{helium f}} = (7228\ \text{J/mol})n_{\text{helium}} = (7228\ \text{J/mol})(6.54\times10^{-3}\ \text{mol}) = 47.3\ \text{J} \approx 47\ \text{J}$$

$$E_{\text{argon f}} = (7228\ \text{J/mol})n_{\text{argon}} = (7228\ \text{J/mol})(1.449\times10^{-2}\ \text{mol}) = 104.7\ \text{J} \approx 105\ \text{J}$$

(c) The amount of heat transferred is

$$E_{\text{helium f}} - E_{\text{helium i}} = 47.3\ \text{J} - 30.4\ \text{J} = 16.9\ \text{J} \qquad E_{\text{argon f}} - E_{\text{argon i}} = 104.7\ \text{J} - 121.6\ \text{J} = -16.9\ \text{J}$$

The helium gains 16.9 J of heat energy and the argon loses 16.9 J. Thus approximately 17 J are transferred from the argon to the helium.

(d) The equilibrium condition for monatomic gases is

$$(e_{\text{helium}})_{\text{avg}} = (e_{\text{argon}})_{\text{avg}} \Rightarrow \frac{E_{\text{helium f}}}{N_{\text{helium}}} = \frac{E_{\text{argon f}}}{N_{\text{argon}}} = \tfrac{3}{2} k_B T_f$$

Substituting the above values,

$$\frac{47.3\ \text{J}}{3.936\times10^{21}} = \frac{104.7\ \text{J}}{8.726\times10^{21}} = \tfrac{3}{2}(1.38\times10^{-23}\ \text{J/K})T_f \Rightarrow T_F = 580\ \text{K} = 307°\text{C}$$

(e) The final pressure of the helium and argon are

$$p_{\text{helium f}} = \frac{N_{\text{helium}} k_B T}{V_{\text{helium}}} = \frac{(3.936\times10^{21})(1.38\times10^{-23}\ \text{J/K})(580\ \text{K})}{100\times10^{-6}\ \text{m}^3} = 3.15\times10^5\ \text{Pa} \approx 3.1\ \text{atm}$$

$$p_{\text{argon f}} = \frac{N_{\text{argon}} k_B T}{V_{\text{argon}}} = \frac{(8.726\times10^{21})(1.38\times10^{-23}\ \text{J/K})(580\ \text{K})}{200\times10^{-6}\ \text{m}^3} = 3.49\times10^5\ \text{Pa} \approx 3.4\ \text{atm}$$

Problem #5: RDK 19.2

Solve: During each cycle, the work done by the engine is $W_{\text{out}} = 200\text{ J}$ and the engine exhausts $Q_C = 400\text{ J}$ of heat energy. By conservation of energy,

$$Q_H = W_{\text{out}} + Q_C = 200\text{ J} + 400\text{ J} = 600\text{ J}$$

Thus, the efficiency of the engine is

$$\eta = \frac{W_{\text{out}}}{Q_H} = \frac{400\text{ J}}{600\text{ J}} = 0.33$$

Problem #6: RDK 19.14

Model: The heat engine follows a closed cycle, starting and ending in the original state. The cycle consists of three individual processes.
Solve: **(a)** The work done by the heat engine per cycle is the area enclosed by the *p*-versus-*V* graph. We get

$$W_{\text{out}} = \tfrac{1}{2}(200\text{ kPa})(100\times10^{-6}\text{ m}^3) = 10\text{ J}$$

The heat energy transferred into the engine is $Q_H = 30\text{ J} + 84\text{ J} = 114\text{ J}$. Because $W_{\text{out}} = Q_H - Q_C$, the heat energy exhausted is

$$Q_C = Q_H - W_{\text{out}} = 114\text{ J} - 10\text{ J} = 104\text{ J} \approx 0.10\text{ kJ}$$

(b) The thermal efficiency of the engine is

$$\eta = \frac{W_{\text{out}}}{Q_H} = \frac{10\text{ J}}{114\text{ J}} = 0.088$$

Assess: Practical engines have thermal efficiencies in the range $\eta \approx 0.1 - 0.4$.

Problem #7: RDK 19.17

Model: The Brayton cycle involves two adiabatic processes and two isobaric processes. The adiabatic processes involve compression and expansion through the turbine.
Solve: The thermal efficiency for the Brayton cycle is $\eta_B = 1 - r_p^{(1-\gamma)/\gamma}$, where $\gamma = C_P/C_V$ and r_p is the pressure ratio. For a diatomic gas $\gamma = 1.4$. For an adiabatic process,

$$p_1V_1^\gamma = p_2V_2^\gamma \quad \Rightarrow \quad p_2/p_1 = (V_1/V_2)^\gamma$$

Because the volume is halved, $V_2 = \frac{1}{2}V_1$ so

$$r_p = p_2/p_1 = (2)^\gamma = 2^{1.4} = 2.639$$

The efficiency is

$$\eta_B = 1 - (2.639)^{-0.4/1.4} = 0.24$$

Problem #8: RDK 19.53

Model: The heat engine follows a closed cycle. For a diatomic gas, $C_V = \frac{5}{2}R$ and $C_P = \frac{7}{2}R$.

Visualize: Please refer to Figure P19.53.

Solve: (a) Since $T_1 = 293$ K, the number of moles of the gas is

$$n = \frac{p_1V_1}{RT_1} = \frac{(0.5\times1.013\times10^5 \text{ Pa})(10\times10^{-6} \text{ m}^3)}{(8.31 \text{ J/mol K})(293 \text{ K})} = 2.08\times10^{-4} \text{ mol}$$

At point 2, $V_2 = 4V_1$ and $p_2 = 3p_1$. The temperature is calculated as follows:

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} \quad\Rightarrow\quad T_2 = \frac{p_2}{p_1}\frac{V_2}{V_1}T_1 = (3)(4)(293 \text{ K}) = 3516 \text{ K}$$

At point 3, $V_3 = V_2 = 4V_1$ and $p_3 = p_1$. The temperature is calculated as before:

$$T_3 = \frac{p_3}{p_1}\frac{V_3}{V_1}T_1 = (1)(4)(293 \text{ K}) = 1172 \text{ K}$$

For process $1 \to 2$, the work done is the area under the p-versus-V curve. That is,

$$W_s = (0.5 \text{ atm})(40 \text{ cm}^3 - 10 \text{ cm}^3) + \tfrac{1}{2}(1.5 \text{ atm} - 0.5 \text{ atm})(40 \text{ cm}^3 - 10 \text{ cm}^3)$$
$$= (30\times10^{-6} \text{ m}^3)(1 \text{ atm})\left(\frac{1.013\times10^5 \text{ Pa}}{1 \text{ atm}}\right) = 3.04 \text{ J}$$

The change in the thermal energy is

$$\Delta E_{th} = nC_V\Delta T = (2.08\times10^{-4} \text{ mol})\tfrac{5}{2}(8.31 \text{ J/mol K})(3516 \text{ K} - 293 \text{ K}) = 13.93 \text{ J}$$

The heat is $Q = W_s + \Delta E_{th} = 16.97$ J. For process $2 \to 3$, the work done is $W_s = 0$ J and

$$Q = \Delta E_{th} = nC_V\Delta T = n\left(\tfrac{5}{2}R\right)(T_3 - T_2)$$
$$= (2.08\times10^{-4} \text{ mol})\tfrac{5}{2}(8.31 \text{ J/mol K})(1172 \text{ K} - 3516 \text{ K}) = -10.13 \text{ J}$$

For process $3 \to 1$,

$$W_s = (0.5 \text{ atm})(10 \text{ cm}^3 - 40 \text{ cm}^3) = (0.5\times1.013\times10^5 \text{ Pa})(-30\times10^{-6} \text{ m}^3) = -1.52 \text{ J}$$
$$\Delta E_{th} = nC_V\Delta T = (2.08\times10^{-4} \text{ mol})\tfrac{5}{2}(8.31 \text{ J/mol K})(293 \text{ K} - 1172 \text{ K}) = -3.80 \text{ J}$$

The heat is $Q = \Delta E_{th} + W_s = -5.32$ J.

	W_s (J)	Q (J)	ΔE_{th}
$1 \to 2$	3.04	16.97	13.93
$2 \to 3$	0	−10.13	−10.13
$3 \to 1$	−1.52	−5.32	−3.80
Net	1.52	1.52	0

(b) The efficiency of the engine is

$$\eta = \frac{W_{net}}{Q_H} = \frac{1.52 \text{ J}}{16.97 \text{ J}} = 0.090 = 9.0\%$$

(c) The power output of the engine is

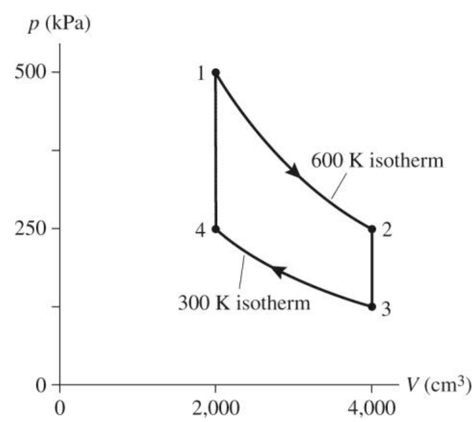
$$500\left(\frac{\text{revolutions}}{\text{min}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{W_{net}}{\text{revolution}}\right) = \left(\frac{500}{60}\right)(1.52 \text{ J/s}) = 13 \text{ W}$$

Assess: Note that more than two significant figures are retained in part (a) because the results are intermediate. For a closed cycle, as expected, $(W_s)_{net} = Q_{net}$ and $(\Delta E_{th})_{net} = 0$ J.

Problem #9: RDK 19.63 (BONUS)

Model: The closed cycle of the heat engine involves the following four processes: isothermal expansion, isochoric cooling, isothermal compression, and isochoric heating. For a monatomic gas $C_V = \frac{3}{2}R$.

Visualize:



Solve: Using the ideal-gas law,

$$p_1 = \frac{nRT_1}{V_1} = \frac{(0.20 \text{ mol})(8.31 \text{ J/mol K})(600 \text{ K})}{2.0\times10^{-3} \text{ m}^3} = 4.986\times10^5 \text{ Pa}$$

At point 2, because of the isothermal conditions, $T_2 = T_1 = 600$ K and

$$p_2 = p_1\frac{V_1}{V_2} = (4.986\times10^5 \text{ Pa})\left(\frac{2.0\times10^{-3} \text{ m}^3}{4.0\times10^{-3} \text{ m}^3}\right) = 2.493\times10^5 \text{ Pa}$$

At point 3, because it is an isochoric process, $V_3 = V_2 = 4000 \text{ cm}^3$ and

$$p_3 = p_2\frac{T_3}{T_2} = (2.493\times10^5 \text{ Pa})\left(\frac{300 \text{ K}}{600 \text{ K}}\right) = 1.247\times10^5 \text{ Pa}$$

Likewise at point 4, $T_4 = T_3 = 300$ K and

$$p_4 = p_3\frac{V_3}{V_4} = (1.247\times10^5 \text{ Pa})\left(\frac{4.0\times10^{-3} \text{ m}^3}{2.0\times10^{-3} \text{ m}^3}\right) = 2.493\times10^5 \text{ Pa}$$

Let us now calculate $W_{net} = W_{1\to2} + W_{2\to3} + W_{3\to4} + W_{4\to1}$. For the isothermal processes,

$$W_{1\to2} = nRT_1\ln\frac{V_2}{V_1} = (0.20 \text{ mol})(8.31 \text{ J/mol K})(600 \text{ K})\ln(2) = 691.2 \text{ J}$$

$$W_{3\to4} = nRT_3\ln\frac{V_4}{V_3} = (0.20 \text{ mol})(8.31 \text{ J/mol K})(300 \text{ K})\ln\left(\tfrac{1}{2}\right) = -345.6 \text{ J}$$

For the isochoric processes, $W_{2\to3} = W_{4\to1} = 0$ J. Thus, the work done per cycle is $W_{net} = 345.6 \text{ J} \approx 350 \text{ J}$. Because $Q = W_S + \Delta E_{th}$,

$$Q_{1\to 2} = W_{1\to 2} + (\Delta E_{\text{th}})_{1\to 2} = 691.2\text{ J} + 0\text{ J} = 691.2\text{ J}$$

For the first isochoric process,

$$Q_{2\to 3} = nC_{\text{V}}\Delta T = (0.20\text{ mol})\left(\tfrac{3}{2}R\right)(T_3 - T_2)$$
$$= (0.20\text{ mol})\tfrac{3}{2}(8.31\text{ J/mol K})(300\text{ K} - 600\text{ K}) = -747.9\text{ K}$$

For the second isothermal process

$$Q_{3\to 4} = W_{3\to 4} + (\Delta E_{\text{th}})_{3\to 4} = -345.6\text{ J} + 0\text{ J} = -345.6\text{ J}$$

For the second isochoric process,

$$Q_{4\to 1} = nC_{\text{V}}\Delta T = n\left(\tfrac{3}{2}R\right)(T_1 - T_4)$$
$$= (0.20\text{ mol})\left(\tfrac{3}{2}\right)(8.31\text{ J/mol K})(600\text{ K} - 300\text{ K}) = 747.9\text{ K}$$

Thus, $Q_{\text{H}} = Q_{1\to 2} + Q_{4\to 1} = 1439.1\text{ J}$. The thermal efficiency of the engine is

$$\eta = \frac{W_{\text{net}}}{Q_{\text{H}}} = \frac{345.6\text{ J}}{1439.1\text{ J}} = 0.24 = 24,$$